Mark Scheme (Result)

October 2020

Pearson Edexcel GCE In A level Further Mathematics

Paper 9FM0/4B

Edexcel and BTEC Qualifications

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- ⌈ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

Qu	Scheme	Marks		AO
1	$H_0 : \sigma_a^2 = \sigma_b^2 \quad H_1 : \sigma_a^2 > \sigma_b^2$	B1		2.5
	$s_a^2 = \frac{145\,496}{6} = [24\,249.33...] \quad s_b^2 = \frac{56\,364.4}{9} = [6262.711...]$	M1		1.1b
	$F_{6,9} = \frac{"24\,249.33..."}{"6262.711..."}$	M1		2.1
	$= 3.872\ldots$	A1		1.1b
	$F_{6,9}$(5% one-tail) c.v. = 3.37	B1		1.1b
	Significant, there is evidence to support Gina's belief	A1		2.2b
		(6)		
		(6 marks)		
	Notes			
	1st B1 for both hypotheses in terms of σ. 1st M1 for at least one of the s^2 calculations correctly attempted NB s_a = 155.72… accept awrt 156 and s_b = 79.137… accept awrt 79.1 2nd M1 for a correct calculation of the test statistic (ft their s^2) 1st A1 for awrt 3.87 2nd B1 for correct cv awrt 3.37 2nd A1 for a correct conclusion mentioning Gina's belief (o.e.)			

Qu	Scheme	Marks	Grade	AO
2(a)	$\bar{x} = 444$	M1	Low	2.1
	$s_x^{\ 2} = \dfrac{1\,577\,314 - 8 \times 444^2}{7} = \dfrac{226}{7} = 32.2857..$	A1	Low	1.1b
	$t_7(5\%)$ 2-tail cv = 2.365	B1	Low	1.1b
	95% CI for μ is: $444 \pm 2.365 \times \sqrt{\dfrac{32.2857...}{8}}$	M1	Med	2.1
	= (439.248…, 448.75…) = awrt **(439, 449)**	A1 (5)	Med	1.1b
(b)	440 is in CI so the average contents statement is OK	B1 (1)	High	2.2b
		(6 marks)		

	Notes
(a)	1st M1 for finding mean and attempting s^2 1st A1 for correct mean and s^2 (accept awrt 3sf) B1 for a correct cv of 2.365 or better 2nd M1 for use of correct formula, ft their mean, s_x and cv for t (use of 1.96 is M0) 2nd A1 for awrt (439,449)
(b)	1st B1 for correct statement about 440 and interval and conclusion

Qu	Scheme	Marks		AO
3 I	(Is feasible as a residual plot but) probably a non-linear relationship	B1		2.2b
	Since the residuals are not randomly scattered about zero	B1		2.4
II	Impossible as a residual plot	B1		2.2a
	Since the residuals do not sum to zero	B1		2.4
III	(Is feasible as a residual plot) and probably a linear relationship	B1		2.2b
	Since the points are randomly scattered about zero	B1		2.4
		(6)		
		(6 marks)		

	Notes
I	1st B1 for stating possibly non-linear (allow a suitable sketch)
	2nd B1 for a suitable comment (e.g. follow a systematic pattern)
II	1st B1 for stating not feasible as a residual plot
	2nd B1 for a correct reason
III	1st B1 for stating probably a linear relationship
	2nd B1 for a suitable supporting reason

Qu	Scheme	Marks		AO
4 (a)	$X \sim B(n, p)$ so $E(X) = np$ and $Y \sim B(m, p)$ so $E(Y) = mp$	M1		3.3
	$E(S) = \dfrac{E(X+Y)}{n+m} = \dfrac{np+mp}{n+m} = p$ so S is unbiased	M1		3.4
	$E(T) = \dfrac{1}{2}\left[\dfrac{E(X)}{n} + \dfrac{E(Y)}{m}\right] = \dfrac{1}{2}\left[\dfrac{np}{n} + \dfrac{mp}{m}\right] = \dfrac{1}{2} \times 2p = p$ so T is unbiased	A1cso		1.1b
		(3)		
(b)	$\text{Var}(S) = \dfrac{np(1-p)+mp(1-p)}{(n+m)^2} = \dfrac{p(1-p)}{n+m}$	M1		2.1
	$\text{Var}(T) = \dfrac{1}{4}\left[\dfrac{np(1-p)}{n^2} + \dfrac{mp(1-p)}{m^2}\right] = \dfrac{p(1-p)(m+n)}{4nm}$	A1		1.1b
	$\text{Var}(S) < \text{Var}(T) \Rightarrow \dfrac{p(1-p)}{n+m} < \dfrac{p(1-p)(m+n)}{4mn} \Leftrightarrow 4mn < (n+m)^2$	M1		1.1b
	$\Leftrightarrow \quad 0 < m^2 + 2mn + n^2 - 4mn \quad \Leftrightarrow \quad 0 < (m-n)^2$ So S always has the smaller variance and is the better estimator	A1cso		2.2a
		(4)	**(7 marks)**	

	Notes
(a)	1st M1 for selecting correct models for X and Y 2nd M1 for using these models to show that either S or T is unbiased A1cso for correctly showing that both are unbiased.
(b)	1st M1 for a correct attempt at Var(S) or Var(T) (need not be simplified) 1st A1 for both correct variances 2nd M1 for a correct inequality in m and n and a first step to clear denominators 2nd A1cso for a correct proof and conclusion

Qu	Scheme	Marks		AO
5(a)	$\int(1-\cos x)\ \mathrm{d}x = [x - \sin x]$	M1		1.1b
	Use of correct limits and $\int \mathrm{f}(x)\ \mathrm{d}x = 1 \Rightarrow 2\pi - 0 - 0 = 1$	M1		1.1b
	so $k = \frac{1}{2\pi}$ (*)	A1*cso		1.1b
		(3)		
(b)	E(X) = π (symmetry) so $\mu = \pi$ so f(μ) = $\frac{1}{2\pi}(1-\cos\pi) = \frac{1}{\pi}$	B1		2.2a
	$\frac{1}{\sigma\sqrt{2\pi}} = "\frac{1}{\pi}"$; so $\sigma = \sqrt{\frac{\pi}{2}}$	M1; A1		1.1b 1.1b
		(3)		
(c)	$\mathrm{P}\left(\frac{\pi}{2} < X < \frac{3\pi}{2}\right) = \frac{1}{2\pi}[x - \sin x]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = \frac{1}{2\pi}\left[\left(\frac{3\pi}{2} - -1\right) - \left(\frac{\pi}{2} - 1\right)\right]$	M1		3.4
	$= \frac{2+\pi}{2\pi}$ (= 0.81830…)	A1		1.1b
	$\mathrm{P}\left(\frac{\pi}{2} < Y < \frac{3\pi}{2}\right) = 0.7899\ldots$	B1		1.1b
	So error is $0.81830... - 0.7899... = 0.0284$	A1		1.1b
		(4)		
		(10 marks)		

	Notes
(a)	1st M1 attempt to integrate (1 – cos$x$) – one correct term 2nd M1 for use of correct limits and correct method for k A1* cso use of $\int \mathrm{f}(x)\ \mathrm{d}x = 1$ seen and no incorrect working seen
(b)	B1 for correctly deducing the value of f(μ) M1 for a correct equation for σ – ft their value for f(μ) [condone for sight of correct g(μ)] A1 for $\sqrt{\frac{\pi}{2}}$ or exact equivalent
(c)	M1 for a correct attempt to find prob – some correct integration and use of limits 1st A1 for a correct answer (exact or 0.818.. or better) B1 for a correct probability from their calculator i.e. 0.7899 or better accept 0.79 2nd A1 for 0.0284 or better

Qu	Scheme	Marks		AO
6(a)	From CI $\bar{x} = \frac{1.193+1.367}{2} = 1.28$	B1		1.1b
	or width = 1.367 – 1.193 = 0.174			
	$1.367 = "1.28" \pm 2.064 \times \frac{s}{\sqrt{25}}$ or "0.174" = $2 \times 2.064 \times \frac{s}{\sqrt{25}}$	M1;A1		3.4 1.1b
	$\Rightarrow s = 0.210755...$	A1		1.1b
	$H_0 : \sigma = 0.175 \quad H_1 : \sigma \neq 0.175$	B1		2.5
	$\chi_{24}^{2} = \frac{24s^2}{\sigma^2} =, \ 34.8092\ldots$ awrt 34.8	M1, A1		3.3 1.1b
	$\chi_{24}^{2}(10\%)$ 2-tail CR is $\chi_{24}^{2} < \underline{13.848}$ or $\chi_{24}^{2} > \underline{36.415}$	B1		2.1
	34.8 is not significant so insufficient evidence that $\sigma \neq 0.175$	A1		2.2b
		(9)		
(b)	$"1.28" \pm z \times \frac{0.175}{\sqrt{25}}$	M1		3.3
	$z = 1.96$	B1		1.1b
	= (1.211…, 1.349…) = awrt **<u>(1.21, 1.35)</u>**	A1		1.1b
		(3)		
		(12 marks)		

	Notes
(a)	1st B1 for finding mean from CI or calculation of width of CI 1st M1 for using the given *t* model to form an equation in *s*. (Allow *t* for 2.064 where $2 < t < 3$) 1st A1 for correct use of $t_{24} = 2.064$ 2nd A1 for $s = 0.21$ or better 2nd B1 for correct hypotheses in terms of σ. 2nd M1 for selecting the appropriate model for this test 3rd A1 for test statistic awrt 34.8 3rd B1 for at least one correct critical value 4th A1 for a correct conclusion confirming that assuming st. dev = 0.175 is OK
(b)	M1 for use of correct formula with $1.6 < z < 2$ (ft $\bar{x}$ if found in (a)) B1 for $z = 1.96$ or better used A1 for an interval awrt (1.21, 1.35)

Qu	Scheme	Marks		AO
7(a)	$X = L_1 + L_2 + L_3 \sim \text{N}\left(594, \sqrt{75}^2\right)$	M1 A1		3.3 1.1b
	$Y = S_1 + ... + S_8 \sim \text{N}\left(592, \sqrt{72}^2\right)$	A1		1.1b
	P($X > Y$) = P($D > 0$) where $D \sim \text{N}\left(2, \sqrt{147}^2\right)$	M1 A1ft		2.1 1.1b
	= 0.56551… awrt **<u>0.566</u>**	A1		3.4
		(6)		
(b)	$W = L - \frac{8}{3}S \Rightarrow W \sim \text{N}\left(\frac{2}{3}, 25 + \frac{64}{9} \times 9\right) = \text{N}\left(\frac{2}{3}, \sqrt{89}^2\right)$	M1 M1,A1		3.3 2.1,1.1b
	P($W > 0$) = 0.528168… awrt **<u>0.528</u>**	M1A1		3.4,1.1b
		(5)		
(c)	$F = L_1 + ... + L_5 \sim \text{N}\left(990, \sqrt{125}^2\right)$	M1 A1		3.1b 1.1b
	P($F < 1000$) = 0.814455… (o.e.)	A1		3.4
	E(cost of Rosa's plan) = $430 \times "0.814..." + 400 \times (1 - "0.814...")$	M1		2.1
	= £ 424.43	A1		1.1b
	Buying 14 small panels cost $14 \times 30 =$ £420 So Rosa's plan is likely to be <u>more expensive</u>	A1		3.2a
		(6)		
		(17 marks)		

Notes

(a)
1st M1 for an attempt at X or Y – expression or implied by one correct distribution
1st A1 for a correct distribution for X <u>or</u> implied by E(D) = 2
2nd A1 for a correct distribution for Y <u>or</u> implied by Var(D) = 147
2nd M1 for a correct strategy – attempt $X - Y$ and P($D > 0$) statement
3rd A1ft for a correct distribution for D ft their X and Y
4th A1 for awrt 0.566

(b)
1st M1 for attempt at a correct model (normal and mean)
2nd M1 for correct expression for variance of their model provided of the form $L - kS$ <u>or</u> $kL - S$
1st A1 for a fully correct distribution
3rd M1 for a correct probability statement using their distribution
2nd A1 for awrt 0.528

(c)
1st M1 for a correct start to solve the problem attempt at F and correct mean
1st A1 for a correct distribution
2nd A1 for using this model to find P($F < 1000$) = awrt 0.814 or P($F > 1000$) = awrt 0.186
2nd M1 for a correct strategy to solve the problem i.e. attempt at expected cost ft their prob
3rd A1 for awrt £424
4th A1 for a correct conclusion must have comparison with £420 and reject Rosa's plan
[(c) is an extended problem and a 3.1, 3.2 question]

Qu	Scheme	Marks	AO
8(a)	$P(\pi X^2 > 10) \Rightarrow P\left(X > \sqrt{\frac{10}{\pi}}\right)$	M1	3.1a
	$= \dfrac{\pi - \sqrt{\frac{10}{\pi}}}{\pi}$	M1	2.1
	= 0.43209… = awrt **0.432**	A1	1.1b
		(3)	
(b)	P(area > median) = 0.5; since (a) < 0.5 therefore **median < 10**	B1	2.2a
		(1)	
(c)	Area of triangle = $0.5x^2\sin x$	M1	3.1a
	$\text{E(area)} = \int_{[0]}^{[\pi]} \frac{1}{\pi}\frac{1}{2}x^2 \sin x \, dx$	M1	1.1b
	$= \frac{1}{2\pi}\int_{[0]}^{[\pi]} x^2 \, d(-\cos x) = \frac{1}{2\pi}\left\{\left[-x^2\cos x\right]_{[0]}^{[\pi]} - \int_{[0]}^{[\pi]} -2x\cos x \, dx\right\}$	M1	2.1
	$= \left\{\left[\frac{-x^2\cos x}{2\pi}\right]_{[0]}^{[\pi]}\right\} + \left[\frac{x\sin x}{\pi}\right]_{[0]}^{[\pi]} - \frac{1}{\pi}\int_{[0]}^{[\pi]} \sin x \, dx$	M1 A1	1.1b 1.1b
	$= \frac{\pi^2}{2\pi} - 0 + 0 - 0 + \left(-\frac{1}{\pi}\right) - \left(\frac{1}{\pi}\right) =, \underline{\frac{\pi}{2} - \frac{2}{\pi}}$	M1 A1	1.1b 1.1b
		(7)	
		(11 marks)	

Notes

(a)	1st M1 reduce the problem to a probability about X 2nd M1 for use of the uniform distribution (a correct expression ft their value 1.784..) A1 for awrt 0.432
(b)	B1 for statement that median < 10 supported by argument about answer to (a) being < 0.5
ALT	Median area is given by $\pi \times \left(\frac{\pi}{2}\right)^2$ = 7.751… < 10 so median < 10
(c)	1st M1 for a correct expression for area in terms of x 2nd M1 for realisation that need to use E(g(X)) formula and a correct expression (ignore limits) 3rd M1 for attempt to use integration by parts 4th M1 for a 2nd use of integration by parts 1st A1 for correct integration (ignore limits) 5th M1 for clear use of the correct limits 2nd A1 for $\frac{\pi}{2} - \frac{2}{\pi}$ [(c) is an extended problem and also involves work from pure for the integration]